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Column Better than blackboard

What's our goal in teaching math to non-math students?

In this column 'Better than blackboard' Nelly Litvak writes about teaching mathematics at university. She will address problems that many university teachers face.

We call it service teaching. Teaching mathematics to non-mathematics students. Physics, chemistry, biotechnology, mechanical engineering, business administration, psychology and innovation sciences... They all need to learn mathematics, there is no question about it. Often this is a responsibility of the mathematics department, fully or partially, depending on the university.

> I am happy to introduce a student illustrator Mara Chelărescu. Mara is the third year Applied Mathematics student at Eindhoven University of Technology. In her drawings, Mara takes inspiration from her favorite animated shows. She has been passionate about illustration and animation for as long as she can remember. Her future goal is to find a way in which she can combine her passions to animation and mathematics. About illustrations for this article, Mara says: "A really big point for me was to showcase women in my drawings since I believe we are already underrepresented in STEM. Additionally, I wanted to make sure to dress them up in cute clothes to show you can both be smart and dress pretty." I couldn't agree more! I hope Mara's cheerful drawings will brighten your day, as they did for me.

For many years I, just like many colleague mathematicians, have been pouring my time and soul into service teaching. Even if the material is basic for us, other challenges are plenty. The classes are large, the courses cover many topics in a short time. And maybe even more importantly, many of these students are not motivated for mathematics itself, they came to study something else. So they get bored, struggle, massively fail the exams. I see this every year and I feel that gradually the situation is getting even worse.

And so lately I caught myself wondering, do we, mathematics teachers, have the right goals of service teaching in our mind? Do we even have well-defined goals besides covering a scary long list of topics in just a couple of weeks? What exactly is our service? And do we serve non-math students well?

What to teach and what to learn

Usually when we think about the goals of a service course, we think in terms of topics. Different programs have different needs, and we do our best to align the topics with the program. However, covering a list of topics is not what I mean by 'the goal'.

To explain what I mean exactly by the goal of a course, I will use an annoying method of business trainers. (A side remark, as a woman in science, I went through tons of trainings, that's why I know). So, the method is called 'Three Why's'. Here is my attempt, feel free to answer the Three Why's differently, but I hope you will agree in general lines. Here we go.

- Students need to learn these topics.
- Why do they need to learn these topics?
- Because they need it in their further study.
- Why do they need their study?
- Because this is how they become a qualified professional.
- Why do they need to be a qualified professional?
- Because then they have the tools to solve complex problems in practice.

Aha. So eventually we need to give the students mathematical tools to solve complex problems in practice. Since recently, for me, this is the goal of my service teaching.

And the beauty is, this is exactly what the students want to learn. I know because I asked. In Figure 1 you see a screenshot from the course Statistics for Mechanical Engineering at the University of Twente. Students had to distribute 100 points over different possible goals. The figure shows the total points for each goal.

Yes, many students just want to pass. This is very typical in our grade-oriented

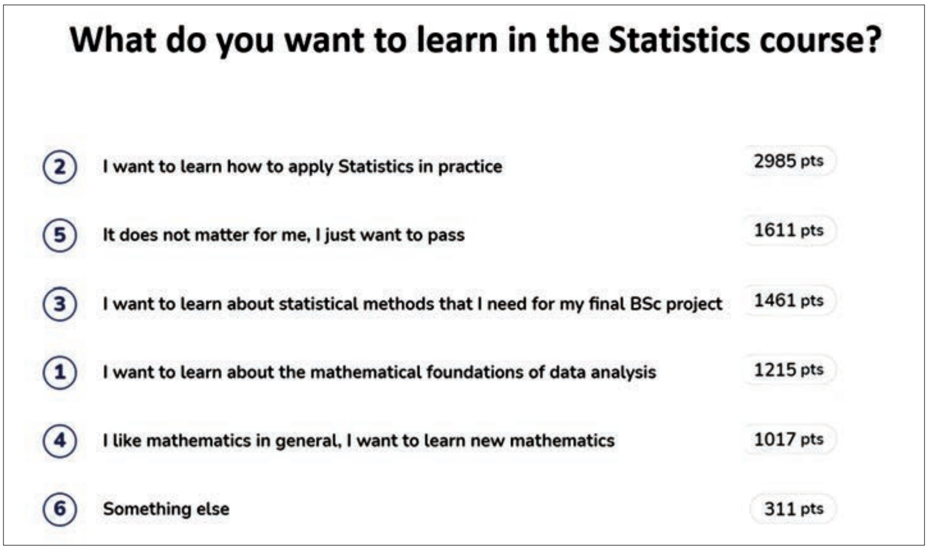

Figure 1 Students distributed 100 points over six possible goals of the course. The figure shows a total score for each goal. Most common goal is to learn how to apply Statistics in practice.

system, and we will come back to it in later articles. Still, the goal “I want to learn how to apply mathematics in my profession”, comes confidently first. Bingo, this is exactly what I want to teach them! Once students and I agree on the common goal, we are more likely to achieve this goal together.

Framing matters

One may say, there is no difference between the high-level goals and the list of topics. Students subscribed to the program, and by doing so they agree on the topics, so we already have common goals.

As a rational logical mathematician, I hear you. However, Daniel Kahneman received the Nobel prize for showing that humans are not rational, and this is what I observed from my experience once I started paying attention. If I have to explain how I feel the difference, I would say, for me the difference is in a mental dialogue with the students. This is more emotional than mathematically precise, but learning is an emotional process. Recently I had a chance to hear a fascinating lecture by educational psychologist André Baars with exactly this title: ‘Learning as an emotional process not a rational one’.

Illustration: Mara Chelărescu

Let me make a humble attempt to demonstrate how setting a goal versus listing topics feels differently for me. I will do this with a mental experiment.

Suppose I open a course with a list of topics. Then my mental dialogue with the students is something like this:

> I: “These are the topics that you will need to learn. Believe me, they are important for you.”

What do you think is the reaction in students’ mind? I don’t have an illusion that even one student thinks: “Oh, yes, confidence intervals! Cannot wait to learn about them!” More realistic reactions are very different.

> Students: “I have no idea about these topics, I hope I can pass.” “I believe you, but I don’t see myself how it is relevant.” “Another math course, I have to go through it.”

I know these sound negative and stand-offish. But what realistic positive reaction do you expect? If you come up with one, please drop me an e-mail.

Now, when my goal is to give the students the tools they need, my conversation is very different.

> I: “What do you want to learn?”
>
> Students: “I want to learn how to use Statistics in my profession.”
>
> I: “Great, this is exactly what I want to teach you!”

This sounds more positive, and this is important. We want the students to invest time and effort in the course, so they must feel that they do something meaningful for them. It is not a magic motivation bullet but at least it doesn’t push the students to the passenger sit from the very start.

Weeding the topics

It is a well-known that once you set a clear goal, things somehow start happening for you. One rational explanation of this mysterious fact is that every day we make lots of choices. Once we have formulated a clear goal, we start making choices towards this goal, consciously and unconsciously.

I see this happening in my service teaching in a very tangible way. Once I decided that I am there for the students to become better professionals, I started dramatically reducing the list of topics. For instance, soon I will teach Statistics for Chemistry. I have only four weeks, and the list of topics is long. I went through these topics with my colleagues in the Chemistry department, and asked, for instance, “Is classical hypothesis testing often used in chemistry?” No? Remove it from the list. I love to explain the type-II error, but tough luck. Kill your babies. Better spend more time on error propagation, which is heavily used in chemistry.

What if one day some unfortunate chemist from my class needs hypothesis testing? I believe that if this chemist understood the reasoning behind error propagation, then they will figure out hypothesis testing easily. Students cannot learn everything in 4 weeks. Something has to go. I stick to the fundamental understanding and to a few topics, most relevant to the students’ specialty. These are the essentials. Learning more than that is a luxury that these students can’t afford. They are very busy learning their own field, which is not mathematics!

Illustration: Mara Chelărescu

Screws are relevant and dices are not
Students' evaluations in service teaching can be very painful. Most stinging part, at least in my experience, is a medium to low score for the relevance.

It is painful because as a mathematician I am thrilled about the power of mathematical abstraction. I explain probability on a dice, and exactly the same theory applies to train parts, coding errors, stock markets and genetic mutations. It is very natural to try to bring this message to the students. Look, these are dices, but it could be screws, grades, or anything you like.

Sadly, this doesn't work. When I taught Statistics for Mechanical Engineering for the first time, long ago, this was the comment: we don't see the relevance, give us mechanical engineering examples. My first reaction was that the students were wrong about it. They were completely missing the most essential translation step! But then I realized that this translation step is most relevant for me, not for them. I am a mathematician, it is my job to see mathematical models everywhere around me, it is the essence of my profession to see how dice and screws are the same thing. However, engineers generally don't care that the same mathematical model applies everywhere. Of course, they are perfectly capable to learn this, but why should they? They have many other things to learn that we, mathematicians, have no idea about. When it comes to mathematics, they want to see how it serves their profession. When we teach them with dice, we basically ask them to do our job.

Alright, now what? How can I come up with relevant examples? Problem is, I have no knowledge in Mechanical Engineering!

And then I had an idea, which I believe was my first serious step in improving my service teaching. Next year at the first class I said: "Look, last year students asked for more examples in Mechanical Engineering. Fair enough, I agree this is useful. However, here is a problem. I cannot come up with these examples because I am not a mechanical engineer. But guess what? You are!"

I gave them a bonus assignment for maximum $+1$ point to the grade, to come up with their own examples for the course material. About 10% of the students did. I included these examples in the lectures (I gave lectures back then, now I don't, see my column 'We shouldn't give classroom lectures anymore' [3].) Next year I did it again. After two years, I had one or two examples for every topic. And they were good! There were examples about companies in the Netherlands, research at our university, local construction works, et cetera.

After that my relevance scores got better. Yet, I constantly had to raise my game. I replaced a reader by a book written for engineers. I included case studies relevant to the project in this quarter. In fall 2019 I recorded a complete set of videos [2]. Each and every example in the videos is related to Mechanical Engineering. By far the majority of the examples came from the students.

As I am writing this article, this comment appeared on YouTube three days ago, on the video about the expectation and the variance of discrete random variables: "Thank you so much for this playlist. It is literally life-saving! This is the best playlist I have come to find so far. Much appreciated!!" This is nice to hear. Yet, now that I have to teach Chemistry students, I will make another playlist. Of course, the mathematical content is exactly the same. As a mathematician I know, one great basic Statistics course is enough for students of all specialties worldwide. This by the way was exactly a comment of a physicist who happened to see me recording. But relevant examples have a function: they make the relevance clear to the students. It matters for them which examples I use. And therefore it matters for me.

Choices in testing
Let me state this right away, "come in through the door", as the Dutch saying goes. Standard written tests don't work in service teaching. These tests generate piles of papers full of unreadable scribbles. Grading this, parsing half a point here and there, takes way too much time and kills my soul. If there is one thing I want to change in our system, it is this: eliminating written tests in the current form, especially for non-mathematics students. My principles are:

– Use automatically graded digital tests whenever possible.
– If a write-up is needed, then assess the quality of writing.
– Use assignments instead of exams to assess application of mathematics in context.

Now I will explain these principles in somewhat more details.

Use automatically graded digital tests whenever possible
In basic math courses, I would go for a fully digital test, graded automatically. It will probably not take long before ChatGPT can grade a basic written test, but before AI comes that far, I will use multiple choice and final answer questions.

Colleagues often say, this way you cannot see the students' way of thinking. It depends on how you set up the test, but I agree, this is mostly correct. Yet, I believe that the write-up produced in a standard written test as we do it now, doesn't add much. Here are my reasons. In most cases, if a student gives the right answer, they had the right reasons. The only added value of a written test is to give partial points to the students who made some right steps but arrived to a wrong answer. However, in this case, the write-up is often very poor. So, the teachers spend tons of time and efforts deciphering scarcely written calculations, and deciding whether a partially right idea is worth 1 or 1.5 points. I am convinced that these extra points don't justify the effort.

Students in my courses do complain about not getting partial points. They say, each question is all or nothing. My answer is, yes, and I find this fair. There are many questions in the test, you need to answer only 50% correctly to pass. If you cannot do this, I believe, you shouldn't pass. I love the binary nature of a digital test. I strongly prefer 50% questions answered correctly than 100% questions answered half correctly.

Partial points reward the students whose knowledge is not very strong, at cost of tremendous amounts of time and decision-making effort from the teachers. Consequently, while teachers are sinking in parsing points, they have no time to develop more effective learning and testing methods. It is truly a vicious circle. I strongly believe that teachers' time is much better spent on effective learning activities and interaction with the students. Then maybe those precious partial points won't matter so much!

If a write-up is needed, then assess the quality of writing

Sometimes a correct write-up is essential. For instance, computer science students need to learn how to write proofs. In the previous column of these series, 'Do we teach what we preach?' [5], Lotte Weedage and I explain how we use Polya's four steps in teaching mathematics students how to write mathematical arguments, and in assessing their writing. For non-math students, Polya's four steps are an overkill. But we can do something similar.

I usually rephrase the Polya's four steps in three steps as follows:

Step 1. Analyse the problem. (Write down what is given, what you need to compute, which mathematical models/statements/methods are relevant.)
Step 2. Solve the problem. (Define the variables, perform the calculations/derivations, explain your arguments.)
Step 3. Analyse the answer. (Does the answer make sense? Why or why not? Is there another way to check the answer? What are the practical implications? Et cetera.)

These three steps sound reasonable to the students. If I have to grade their work, I give 40% for Step 1, 40% for Step 2, and 20% for Step 3. I give 0 for Step 3 if a student writes "The answer is reasonable" but doesn't explain why. I reduce many points in Step 2 if arguments are not explained, or when definitions of the variables are missing, incomplete, or inconsistent. This way the students know that I take writing seriously.

The three steps require a lot of writing from the students, therefore I believe that for non-math students, we can demand such extensive write-up only for a few problems. In my Statistics course for the third year BSc Mechanical Engineering, I asked students to use the three steps only in two Case Studies (three problems each), and in the Final Assignment (I will tell more about this assignment below). The students mostly didn't mind the detailed write-up because it was required only when the problem was framed as a project. The Mechanical Engineering program has many projects with project reports. So, for these students, it was natural to write a small report in Statistics.

Another way to assess the quality of writing, is to set clear and simple criteria that tell the students what a good write-up is. For instance, in the course 'Probability and Statistics' for the first year BSc Computer Science, at Eindhoven University of Technology, we will use the following criteria:

Criterion 1. The problem is solved correctly.
Criterion 2. All variables are defined.
Criterion 3. All steps are explained.

In this course, we give full points only if all three criteria are met, and partial points if two criteria are met. Again, we don't require this for all problems. We chose three topics where a derivation or a proof is essential, and we ask open questions only for these three topics. The rest of the topics are tested digitally with multiple choice and final answer questions. The course is

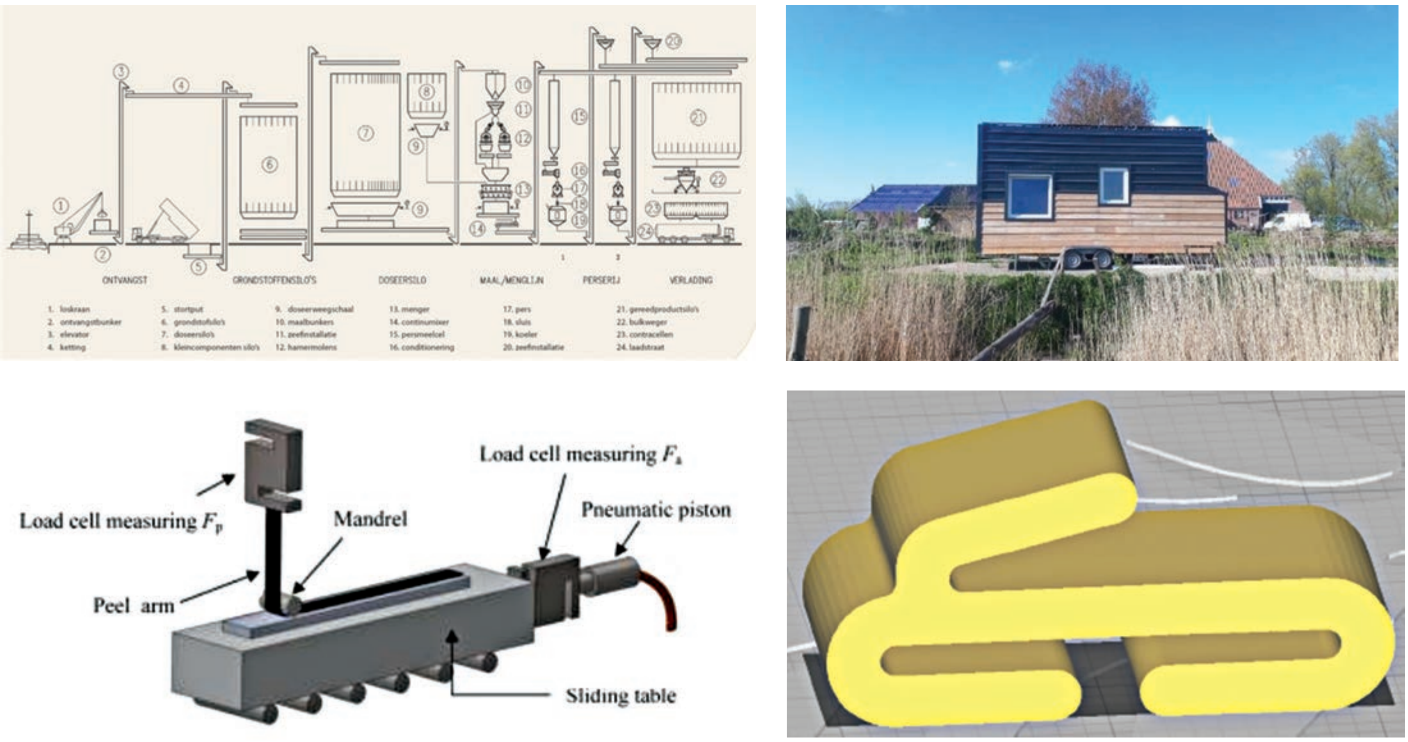

Figure 2 Examples of a Final Assignment, Statistics for Mechanical Engineering. Top left: factory for producing animal feed. Top right: building a tiny house. Bottom left: equipment for removing medical glue. Bottom right: a 3D printed part for holding a medical mask on a ski helmet.

running right now, and I will tell in later articles how this worked out.

Use assignments instead of exams to assess application of mathematics in context
As I said above, the goal of my Statistics course for Mechanical Engineering is to teach the students how to apply Statistics in their profession. Then I quickly realized that the standard test, written or digital, will not work for assessing this goal. In a test, even if a problem is based on a case from Mechanical Engineering, it is already formulated as statistics problem. In my experience, based on the many-many tutorials, when students see such problem, they don't think about how the questions is related to the context. Instead, they think: "Which formula should I use?" But this is not what I want to teach and not what they want to learn. I wish, they think: "How can I apply Statistics in this situation?"

And then I remembered my bonus assignment, where I asked the students to come up with their own engineering examples. Some of them wrote that they liked this assignment, and it helped them to understand the material. So, from 2021, I replaced the exam with a final assignment. Students have to find their own cases, formulate statistics problems, solve them, and submit the solution in written in Steps 1,2,3. Of course, I had to make sure that the self-made problems have sufficient coverage and difficulty level. I will not explain all details here, but my co-teacher Fulya Kula and I wrote an article about it [4], you can find all details there. Or, just feel free to reach out, I will be happy to tell you more.

Not all students love the assignment, but we received many very positive responses. And students' creativity and the variety of topics never seize to amaze me! Some examples are in Figure 2.

The drawback of the assignment is that each assignment is a separate project, and it takes a lot of time to grade. In the last edition of the course, Fulya Kula and I tried to mitigate this by adding a group part and reducing an individual part. Even then the grading takes a lot of time. On the bright side, this grading is very interesting because of the variate examples and detailed solutions. And if a student failed the assignment, they can improve it based on our feedback and resubmit once more. So neither ours nor students' work is wasted.

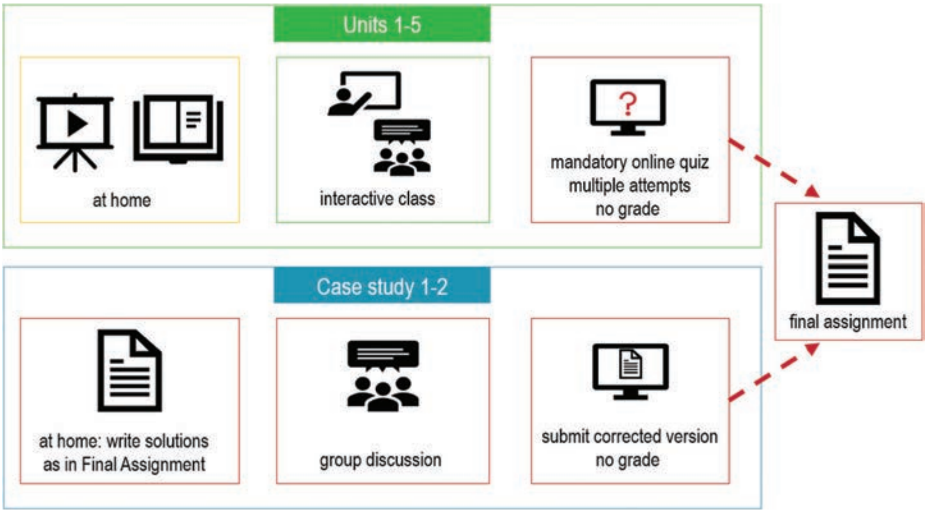

Figure 3 The scheme of the course 'Statistics for Mechanical Engineering', University of Twente.

I believe that assignments on self-chosen topics are a very good way to test students' ability to apply the theory to practice. When students have a choice, they are more in control and have more agency in their study. And we get an unprecedented collection of great examples, and another evidence how fantastically talented and creative our students are!

Choices in teaching
I love the quote from education expert Tim Fawns: "Teachers should shift their focus form generating materials to organizing activities." When designing a course, we always have to ask ourselves: what the students will do? Here is what I believe students should do:

- Learn the theory (read, watch, listen, ask questions, answer questions).
- Practice in the same form as in the exam, and get feedback.

Practicing for exam may sound grade-oriented, but if our exam is meaningful, then practicing for it is meaningful, too. Plus, self-testing is a very effective way of learning.

Figure 3 shows the scheme of my course Statistics for Mechanical Engineering. I use this kind of scheme in most of my courses, you might even recognize it if you read my previous article [5].

The upper (green) row is for learning the theory. The material is divided in five units, and there is one class for each unit. Before a class, the students watch videos and read a book. During the class, I use online quiz about theory with questions that provoke errors and discussion. Then I explain the solution. In Figure 4 you can see my favourite question on hypothesis testing. The topic is conceptually difficult, and it shows! When confronted with such questions, students realize what they don't know, make mistakes in a safe way, and immediately hear what they did wrong. All these are is very useful for learning [1].

After the class, students make a mandatory quiz at home, without a grade but

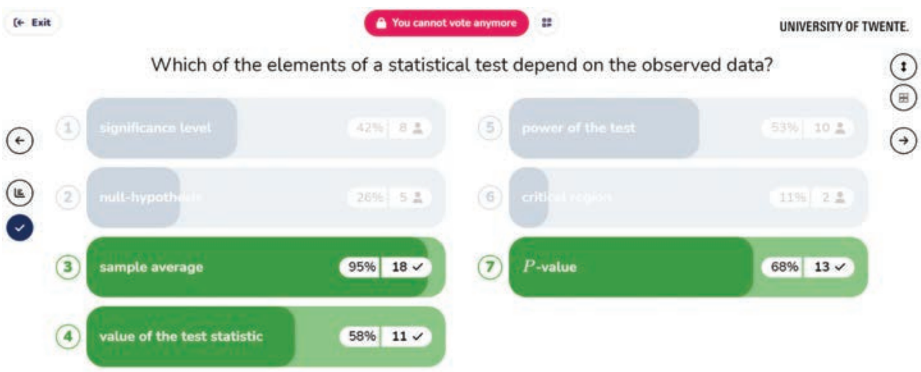

Figure 4 A question on hypothesis testing,

Illustration: Mara Chelărescu

with a high minimal score requirement. I use standard problems with randomized numbers, so a student cannot just copy a numerical answer. If they want to copy then at least they must copy a script or a formula and use it themselves.

The attendance of interactive classes is variable but generally higher than that of lectures.

The bottom (blue) row is where students practice to write the solution in three steps as we require in the Final Assignment. We offer two Case Studies, based on the project of that quarter, each consisting of three problems. The students write solutions at home. In class, they work in groups, compare their solutions to each other and make corrections. We walk around and give feedback on their solutions and write-up. After the discussion, they can see our solutions in the same three steps. Case Studies have no grade but it is mandatory to submit the three steps for all problems, and this is checked by a student assistant. Importantly, students practice the three steps and compare their work to our solutions in this form throughout the course material.

I insist that, if we require write-up at the exam, then students must practice write-up in a structured way, and get feedback. Showing how to do it is useful but insufficient. The students will not learn how to write by watching the teacher writing! I hope one day Programme Committees will disallow open written questions if the course doesn't assume structured write-up practice with feedback.

If the test is fully digital, there is no need to practice write-up. Then, in the bottom row it is enough to give many practice tests with automatic feedback, and include opportunities to ask individual questions (tutorials, Q&A's, office hours).

Teaching proper mathematics?

Many times I heard my colleagues saying that math courses must be taught by mathematicians because we understand the nuances of mathematics. We can teach mathematics properly. I agree with this. Even basic mathematics must be explained with rigor. This is the essence of mathematics.

But sometimes I feel we sound like a Michelin chef who wants to give all their customers a proper dining experience, in no less than five courses with sophisticated taste palette. This is fine if our guest wants to spend their evening enjoying haut cuisine. But what if our guest wants to go to bed early today, and run a marathon tomorrow? Then all they need is a quick serving of a simple nutritious food. I am sure that Michelin chef can cook a yummy healthy meal much better than the runner can do at home. So why don't we do just that?

We call this service teaching. So, let's not judge our future biologists, psychologies, engineers and policy makers by their ability to appreciate fine abstract mathematics. Let's serve them the mathematics that they need.

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