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Valedictory Lecture

Mission possible – mathematics

On 23 June 2023, Wil Schilders, professor of Scientific Computing for Industry at Eindhoven University of Technology, delivered his valedictory lecture. In his lecture, he looked back on his academic and industrial career, and especially his mission to promote mathematics as a discipline that has invisible contributions to visible successes, particularly concerning societal and industrial challenges. Mathematics is everywhere, but often one only sees the tip of the mathematical iceberg. Looking forward, with the emergence of high performance computing, data science and artificial intelligence, these are exciting times for mathematicians, necessitating the development of entirely new methods and preparing a rapidly growing population of talented young students and researchers to work on the challenges of our times. The world is becoming extremely complex; mastering this complexity can only be done with the help of mathematicians in close collaboration with colleagues from other sciences. The aforementioned mission is absolutely possible, and the lecture discussed various ways on how to achieve it.

Almost precisely twenty years ago, I gave my inaugural speech in the same room. At that time in Dutch, with the title ‘De wiskundige leest in bedrijf’. The title had a double meaning. Interpreting the word ‘leest’ as a verb, it means that a mathematician reads what is happening in industry, and acts to see whether there are interesting topics of research. Interpreting ‘leest’ as a noun, it means a shoemaker’s tool, and we have a saying in Dutch that, when translated, reads: shoemaker, stick to your last. In other words: stick to what you have been trained for. Between the lines it was also an advice to researchers and engineers in other scientific disciplines: leave the mathematics to the mathematicians, they are the experts, know about the latest developments, have a particular way of thinking that helps in solving complex problems. I became very much convinced about this attitude when working in industry, from 1980 till 2010. Often, you see ad hoc solutions being used, whereas a deeper insight into the cause of the problems should be the first priority. Only when we understand the real cause, we can start to think about solutions. Often, I compare this way of working with that of a doctor. Suppose someone with headaches every day goes to a doctor. One doctor would say: “Take this pill every day, and the headaches will disappear (for the day).” Another doctor could say: “Please sit down and let’s analyse your current situation”, eventually leading to a sound diagnosis and a correct treatment.

Since the introductory lecture in October 2003, a lot has happened. I left industry in 2010, and was fortunate to build a large network of academic and industrial colleagues from all over the world. Gradually, the mission of promoting the use of mathematics for complex challenges was shaping, owing to my position at this great university in the vicinity of the booming industrial area Brainport as well as via the position within the newly created Dutch Platform for Mathematics and positions in ECMI, EU-MATHS-IN and ICIAM. Initially, it looked like a mission impossible, referring to the great series that we watched in the sixties and early seventies, with Peter Graves starring as Jim Phelps, director of the Impossible Missions Force.

Martijn Anthonissen

Wil Schilders

for a better world

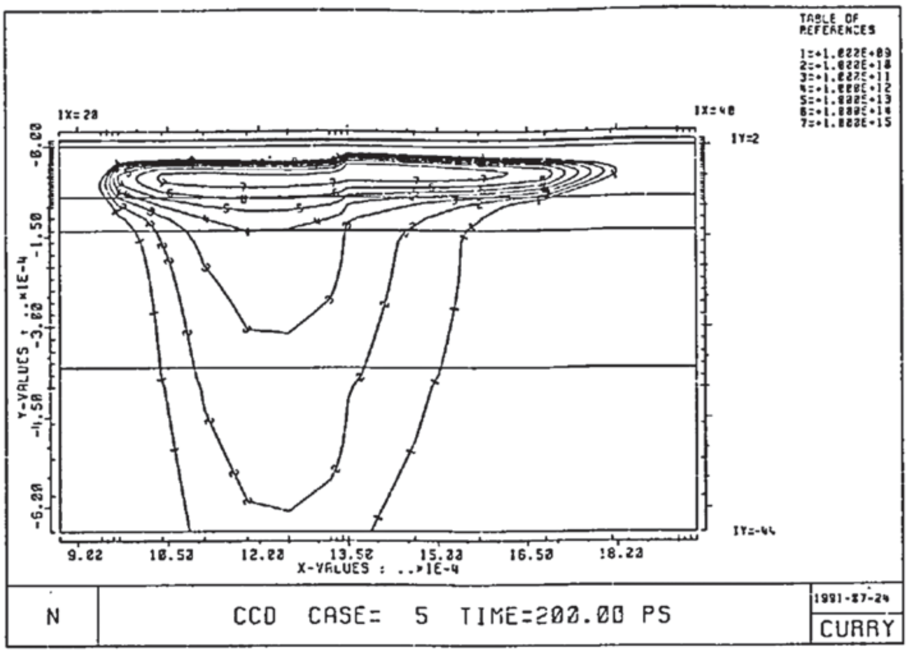
Despite the title of the series, the mission thought to be impossible always turned out to be possible due to smart and intelligent acting. In my opinion, the same is true for the mission of using mathematics to address the complex challenges of our times, in industry, society and other sciences. Clearly, this should be teamwork, with colleagues from other disciplines who know the context and the backgrounds of the challenges. In this way, together we can create a better world with the use of mathematics. This is what I would like to discuss in this valedictory lecture, telling you about my experiences, findings over the years and my view on the future of our mission.

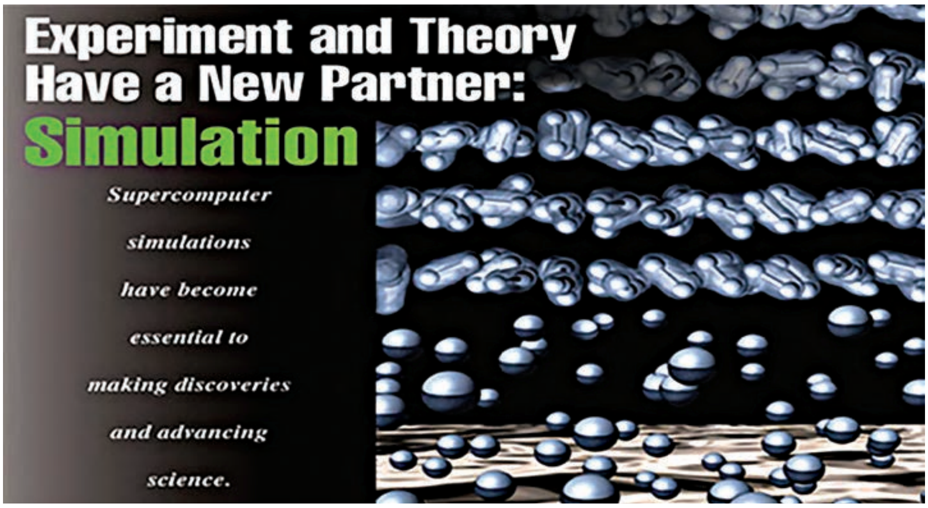
The value of mathematics

When I started to work at Philips Research in 1980, times were much different from this day and age. Software packages as we know them now, able to perform simulations for a wide variety of different structures and designs, did not exist. Instead, for semiconductor device simulation, engineers had made separate software for one-dimensional diodes and one-dimensional transistors. The Mathematical Software Group set out to change this situation, and produce software that could cope with arbitrary semiconductor devices in two dimensions. Engineers at Philips Research thought we were out of our minds, this was absolutely a mission impossible according to them. But we did it, by developing robust and efficient mathematical methods that could cope with a wide variety of devices. The mission was accomplished within a few years and entirely due to the sound mathematical methods.

It was the time when software packages started to be developed in more places, mostly within industry and for a variety of problems: electromagnetic, mechanical, electronic circuits. This development put a high demand on the available mathematics, and new mathematical methods needed to be developed so as to cope with the high demands of these software products. More and more, designing was done behind a computer screen rather than by building prototypes and testing these. It led to the emergence of the so-called third discipline, 'simulation', besides the traditional disciplines 'experiment' and 'theory'.

All of this is made possible by virtue of a so-called mathematical model of the underlying problem, describing its behaviour in detail, often by means of a rather complicated set of differential equations. Clearly,





such models are made in very close collaboration with scientists that know the details of the underlying problem. Once the mathematical model is known, numerical methods can be developed that will simulate the model accurately. This requires bright mathematical minds, and especially the semiconductor device problem caused quite a few headaches before a suitable set of algorithms had been developed. We did this in close collaboration with companies like IBM and AT&T Bell Labs, as well as with Stanford University.

The developments regarding simulations have accelerated since the 1980s, and nowadays a lot of software is available to perform simulations in many domains of science and industry. Designing is done behind a screen, with the aid of virtual design environments. Mathematics plays a crucial role within this evolving area of Computational Science and Engineering, and nowadays mathematical methods are indispensable in the areas of Digital Twinning, Artificial Intelligence, Energy Transition, High Performance Computing, Biomedicine and many more. Lex Schrijver, esteemed colleague from the Centre of Mathematics and Computer Science in Amsterdam, formulated it once in a very nice way:

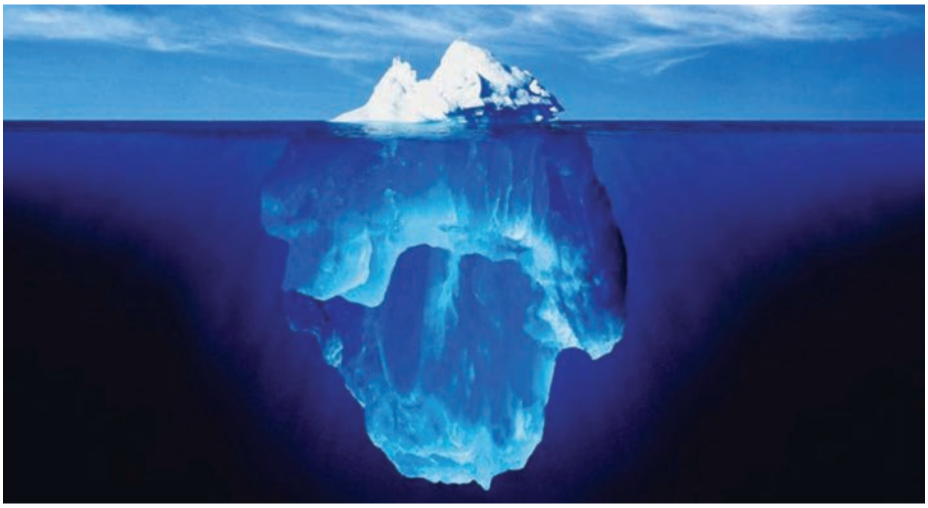
> "Mathematics is like oxygen. You take no notice of it when it's there — if it wasn't, you'd realize you cannot do without it."

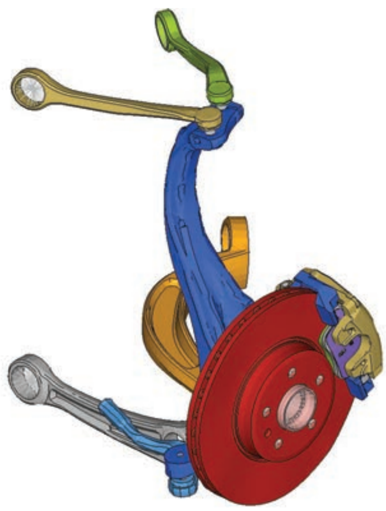
Often, people do not realize how much of the mathematics needed to solve a problem or address a challenge is under the surface. For this reason, I often use the terminology of 'mathematical iceberg'.

Indeed, often the mathematics is invisible, as it is hidden deep in the software package. The success usually comes from applications of the software, i.e. novel designs. Within Philips, engineers were able to design entirely new devices, such as CCD devices for digital cameras, and analyse their behaviour thoroughly using the software containing the robust, fast and efficient mathematical machinery. So the statement is justified that *mathematics provides an invisible contribution to visible success*.

A really great example, showing the power of sound mathematics, is the analysis of disc break squeal by my very valued colleague Volker Mehrmann from TU Berlin. Disc brake squeal is a frequent and annoying phenomenon, and the automotive industry has been trying for decades to reduce squeal by changing the design of the brake and the disc. In 2015, detailed mathematical analysis revealed bifurcations because the designs contained highly stiff springs used to avoid rigid connections. The negative effects of this design technique came as a surprise to the industrial partners! All car manufacturers are extremely interested in the results. Clearly, these hidden properties of the underlying system can only be revealed by mathematicians, it is their specialty!

But there is much more to tell about invisible contributions of mathematics. Everyone knows Moore's Law, which for more than 50 years predicted that every 18 months the speed and density of transistors is doubled. Translated into practical terms it means that our computers have become much faster all the time. Many people then draw the conclusion that the impressive simulations that we see nowadays are entirely due to the greatly improved power of computers. However, this is only half the truth. The mathematics that is hidden in the software packages enabling the simulations is outperforming the hardware improvements, often in a consid-

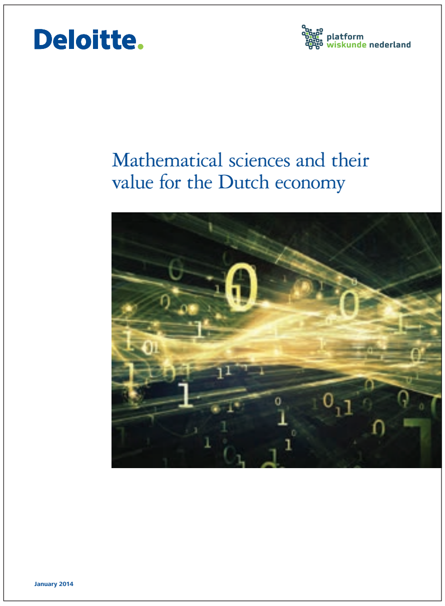


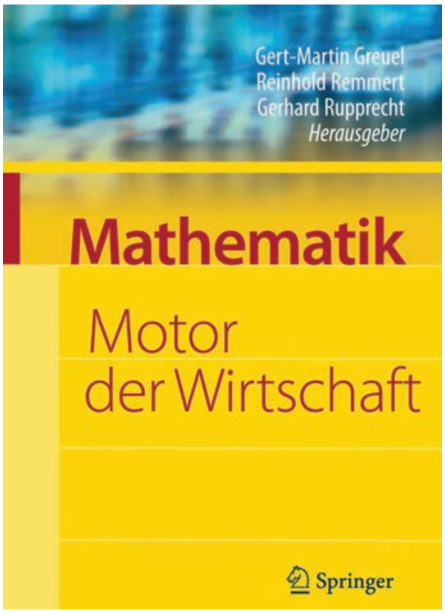


Mathematical method	Period (years)	Improvement hardware	Improvement mathware
Solving large linear systems	35	10.000.000	10.000.000
Linear programming	16	1600	3300
Mixed integer programming	25	6500	870.000
Particle simulations	40	100.000.000	1.000.000.000

erable way. See the table above, where we summarize the improvements for a number of mathematical methods.

The difference is most pronounced in the case of mixed integer programming: if we would rely only on the performance improvements of computers, simulations that would cost an estimated 180 years



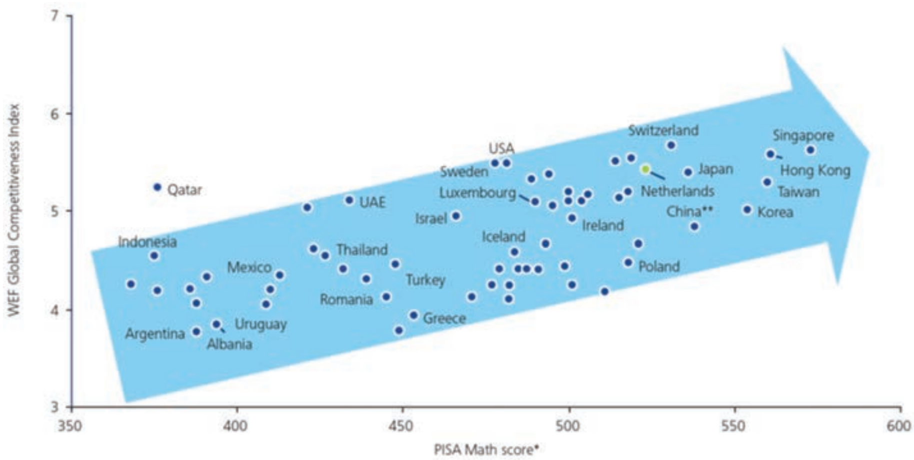


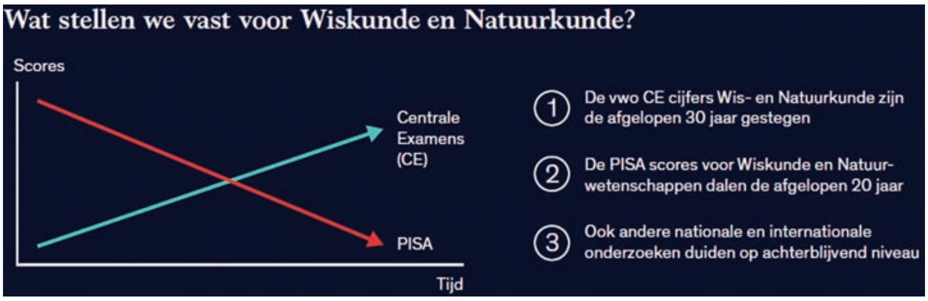
25 years ago, now require 10 days. But if we take the improvements in the mathematical methods also into account, then a simulation is reduced to 1 second. Philippe Toint, emeritus professor in numerical optimisation at Namur University, once said: “I would rather use today’s algorithms on yesterday’s computers than vice versa.” Clearly, if we would only rely on improvements in hardware, we would now be doing the simulations of the 1990s.

In 2014, we asked Deloitte to investigate the value of the mathematical for the Dutch economy, and this led to some stunning conclusions. One of these is that mathematical sciences are contributing for up to 26% to total employment. Because these are high income jobs, the economic contribution of mathematical sciences is even higher, representing around 30% of Dutch national income. Another conclusion is that a strong mathematical sciences foundation is critical to the success of any advanced economy. Better mathematical skills correlate with a more competitive economy and a higher standard of living. Moreover, with the revolutions in computational science, big data, statistics and business analytics the importance of mathematical sciences to society is likely to increase substantially in the coming decades. These revolutions are driven by ever more powerful computers, the data explosion, and improved algorithms.

Similar reports were made in the UK, France and Spain, resulting in similar conclusions. In Germany, a great book was produced in 2008 during the national ‘Year of Mathematics’ entitled *Mathematics: Engine of the Economy*. Twenty leading companies in Germany contributed to this volume. In the introduction, the following statement is made: “Mathematics is everywhere. In the past, it was only individuals who clearly recognised this, such as Alexander von Humboldt with his timeless sentence ‘Mathematical studies are the soul of all industrial progress’ or Werner von Siemens, who gave his brother Wilhelm the urgent advice, ‘Your main course of study must now be mathematics’. Today, there is a consensus that without mathematics, people cannot organise their everyday lives.”

As stated in the Deloitte report, there is a clear correlation between mathematical ability, measured in terms of the Pisa score in secondary schools, and the competitiveness of a country. This is clearly demonstrated in the figure below. Rather worrying in this respect is that in a recent report of McKinsey, it is concluded that the PISA score in The Netherlands is going down in the past 20 years. Clearly, action must be taken to counter this development. The committee on Education within the Dutch Platform for Mathematics is expressing its concern to the ministers, and the message is quite clear: “The trend identified by McKinsey seems to be part of the flattening that is taking place in education on many fronts. There are a complex of causes for this. The student population is changing, qualified teachers are hard to





find, and teaching time for mathematics has been reduced with the introduction of the second phase. Education has to adapt to changes in society, but the impoverishment that is now occurring has negative consequences for the personal development and socialisation of our pupils and for the level at which they enter further education. In STEM studies, they fall short of foreign students in knowledge and work attitude, which weakens the innovative power of our country.”

Let’s get back into a more positive mood again! It is clearly important to convince the general public, secondary school pupils, policy makers, scientists and industry of the value of mathematics for our society. There are many ways to do this, such as with the Deloitte report. Very successful was also the booklet written by Ionica Smeets and Bennie Mols, on behalf of the Dutch Platform for Mathematics, entitled *Success formulas*. It contains 36 stories about applications of mathematics, and a number of interviews with captains of industry and policy makers, to show the versatility of mathematics. Many do not know that mathematics is present everywhere in our society, and this booklet gives an impression of this omnipresence. Some 20 000 copies were distributed all over the country, and in 2021 we made a German version with many new stories.

Scientific Computing for Industry

Over the years, I have been able to demonstrate the value of mathematics especially in industry. When appointed at TU/e, the assignment was ‘Scientific Computing for Industry’, fitting perfectly with the profile I had in mind. Scientific Computing, also referred to as Numerical Analysis, is a discipline that emerged when computers started to become more powerful, say since the 1950s or 1960s. Initially, it was a field that was occupied with a sound analysis of rounding errors, as computers can only use a limited number of digits to represent numbers, and in certain processes this can lead to a considerable build-up of errors.

When computers became more powerful, and software packages started to be developed, the discipline of Computational Science and Engineering started to emerge, and within the area of Scientific Computing, methods needed to be developed that could cope with problems becoming larger rapidly. In the early 1980s the size of problems was around 1000, but later on this grew to tens of thousands, millions and in recent years even billions of variables. Especially important is then to develop methods to solve large linear systems of equations, often with a special structure. We were very fortunate that The Netherlands was leading in this field, and especially the work of Henk van der Vorst and Koos Meijerink on the so-called ICCG method was revolutionary. We were the first to implement this method, developed around 1977, in industrial software, and it enabled us to solve much larger problems than our competitors, and also much faster.

I very much liked the work on numerical linear algebra, and when at Philips, we had extensive discussions on special systems of equations with our team of mathematical advisers: Bram van der Sluis, Henk van der Vorst and Piet Hemker. We became interested in solving more difficult sets of equations, so-called indefinite linear systems occurring in the area of electronic circuit simulation. It led to an intense col-

laboration with Andy Wathen of Oxford University in the early years of this century, and in the end a factorisation method for indefinite matrices was developed. The Oxford group named the method *Schilders' factorisation*, and I was clearly very honoured by this.

The factorisation developed is, in fact, one of the many methods that I developed, together with colleagues and students of course, constituting a red thread over several decades: *mimetic methods*. Already during my PhD at Trinity College Dublin, we worked on methods that used information about the underlying problem. The exponential behaviour of solutions in singularly perturbed problems was used to developed so-called exponentially fitted finite difference schemes. They performed in a fantastic way in practical situations, and had errors that did not depend on the small parameter. Later, at Philips, it turned out that the set of semiconductor device equations was also singularly perturbed, implying that we could use exponentially fitted schemes to discretise the system. The decomposition for indefinite matrices also used information about the underlying problem, namely the fact that there were two different types of variable in the problem. It led to the use of 1×1 and 2×2 blocks in the decomposition process, and led to a great factorisation technique.

I strongly believe in mimetic methods, sometimes also referred to as structure preserving methods. And I think we should adapt the curricula in mathematics to include such methods, as they are extremely important in practice. In most universities, however, scientific computing is taught by presenting rather general methods based on simple Taylor series expansions, using no knowledge at all about the underlying problem. It leads to the violation of physical principles, such as conservation of mass and energy, and to less accurate solutions often requiring much more computation time. During my guest professorship at Bergische Universität Wuppertal (BUW) in 2020–2021, I gave a series of lectures on mimetic methods, showing that these do not only occur within the area of discretisation, but also within the areas of solving linear and nonlinear problems, meshing, model order reduction and more. Together with my esteemed colleagues from BUW, I am writing a book on mimetic methods.

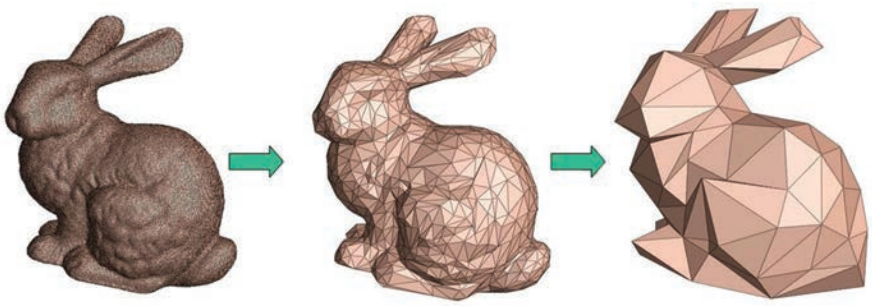
A very successful example of an industrial project solved with mimetic methods was the ASIVA14 project together with Mentor Graphics (MG). One of the problems was that solving nearly periodic electronic circuits could take several weeks, and MG wanted us to develop methods that could considerably reduce this computation time. On investigating this problem, we found that there was a small part of the circuit where much higher frequencies played a role, resulting in the extremely small time steps that lead to high computation times. The idea came up to develop an extremely accurate model for this small high frequent part of the circuit, so that the model could be used in the simulation, and time steps could be used corresponding to the lower frequency part of the circuit. It turned out to be a fantastic idea, leading to speedups of more than a factor of 100 and simulation times of hours rather than weeks.

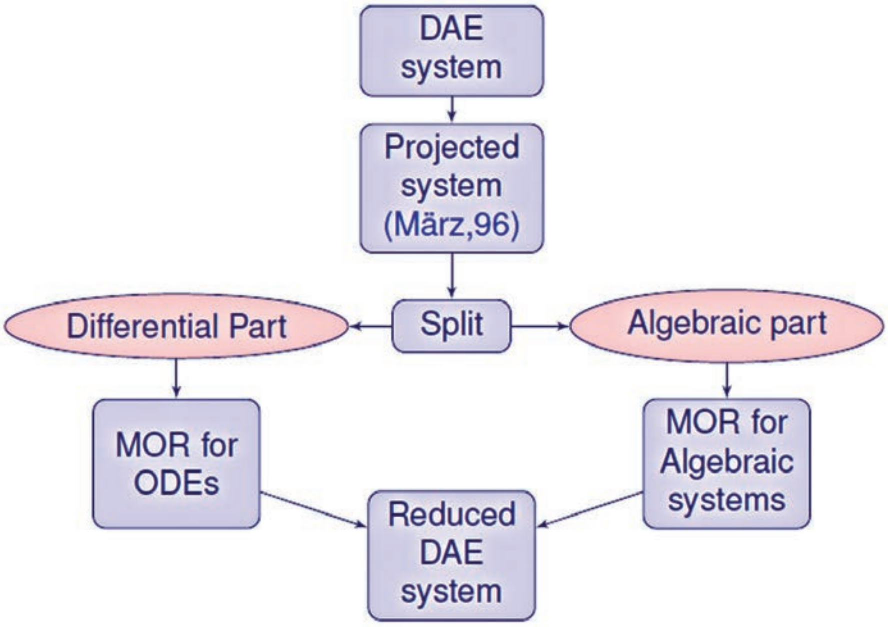
Another successful mimetic method we developed already had its origins within Philips. For strongly nonlinear problems, the well-known Newton's method can often lead to an enormous number of iterations, if it converges at all. The method we developed for semiconductor device simulation is extremely effective, and can lead to a very significant reduction of iterations. This is achieved by employing a nonlinear transformation of variables, making use of knowledge about the nonlinear character.

Mimetic principles also played a decisive role within the area of research that I adopted at the start of my career at TU/e, and have been working on since then: *model order reduction*. Already quite soon I started to use the figure below to explain model order reduction to the general public: It clearly indicates that it is not necessary to know all details, one can often delete superfluous information and concentrate on the dominant behaviour: one will immediately recognize the right picture to be a bunny, one does not need all information as contained in the most left picture.

Model order reduction is on the one hand a flourishing area of research with many researchers contributing to its development, on the other hand it is one of the most important sets of methods used for the efficient solution of problems in industry and, more generally, in the area of computational science and engineering. With the advent of high performance computing, and the growing power of computers, it is very tempting to simulate larger and larger problems. However, the counter side is that the energy consumption will be enormous for all these simulations. Here the adagio 'think twice, compute once' is absolutely valid. Using model order reduction, problems can be reduced considerably in size, but by retaining accuracy of solutions. Model order reduction extracts the dominant features of solutions, which is perfectly adequate to analyse problems, produce designs and investigate processes. So I am making a case here to make much more use of all the advances made in the field of model order reduction over the years, before starting any simulations. Together with Peter Benner, an authority in the world of MOR, I set up the European Model Reduction Network in 2014, and during four years we gathered a large group of more than 300 European researchers working in the field, had many interesting workshops and in the end produced the (open access) *Handbook of Model Reduction* consisting of three volumes that is considered a standard work in the field now.

Clearly, in view of the foregoing, our interest was mainly in mimetic model order reduction. We started out by developing several structure-preserving model reduction methods, following also the work of Roland Freund, viz. his method SPRIM where the 'S' and 'P' stand for 'structure preserving'. At a later stage, we got interested in differential-algebraic equations (DAE),





stimulated by the fact that electronic circuits generate systems of equations of that type. The main idea we had was to split the DAE into a differential and an algebraic part, then use one of the many MOR methods to reduce the differential system, and use other reduction methods for the algebraic part. In this way, we developed the so-called Index Preserving Model Order Reduction methods, IMOR and IIMOR. Indeed, the so-called index of the problem was automatically conserved, due to the procedure followed:

Our work on model order reduction received quite some interest from industry, and led to projects with Mentor Graphics, Siemens, austriamicrosystems, MAGWEL, NXP Semiconductors and Signify. Due to the early workshop in the Lorentz Center in 2005 and the European network EU-MORNET, TU Eindhoven was very visible in this area, leading to much interest from industry. Currently, we are involved in projects that aim at developing compact thermo-mechanical models for electronic circuits, and compact models for LEDs, with model order reduction playing a major role.

Scientific Computing in the future

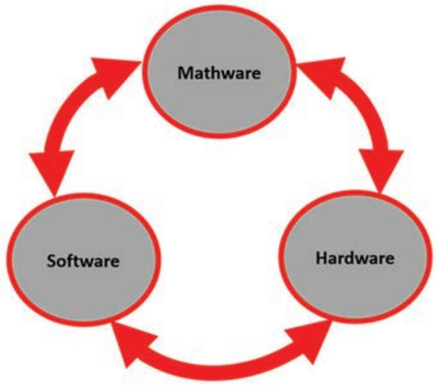
A few years ago, I was a bit pessimistic about the future of Scientific Computing. In the area of numerical linear algebra, I did not see any ground-breaking new developments that would outperform the ICCG method and multigrid methods already developed in the previous century. Some work was done on new preconditioners for special systems of equations, but most of the work was a small improvement of already existing methods. Also in other areas of scientific computing, no revolutionary new methods came up, most of the research was of an incremental nature.

But then, in recent years, the situation changed completely. First of all, *high performance computing* (HPC) became very popular again. In the 1990s, people had worked on HPC, but Moore's Law was still valid, implying that computers were becoming faster all the time. Hence, there was not a real need for HPC methods. Only recently, this changed dramatically. Moore's Law has come to an end, transistors do not become faster anymore, and hence the only way to speed up computing systems is to rely on parallel computing (or quantum computing, but this is so far still a promise) . Supercomputers have become much more mainstream now, and indeed, if we wish to speed up our computations, we will need to resort to parallel machines with many processors as well as GPUs. For the field of scientific computing, this means that we need to concentrate our efforts on methods that are inherently parallel. This is a big challenge for researchers in the scientific computing area. The famous ICCG method that performs so very well on serial computers, performs extremely badly on supercomputers, as can be seen from the table below.

As you can see in the column on the right, ICCG only achieves some 2% of peak performance. This is extremely bad, and means that researchers in numerical linear algebra have a big challenge ahead of them, to develop preconditioners that can bring the performance to a much higher level. So far, I have not seen solutions. Also in other areas of scientific computing, the growing use of supercomputers implies having to work on novel methods. Possibly, entirely new ideas need to be developed in order to cope with these challenges. It also means that the work of mathematicians in the area of computational science and engineering (CSE) is becoming extremely important. However, often people only speak about software and hardware, and the question then is: where is the mathematics? Hidden in the software? Clearly, it is important to first think carefully about the mathematical methods before any software can be written. For this reason,

HPCG Benchmark June 2019

Rank	Site	Computer	Cores	HPL Rmax (Pflop/s)	TOP500 Rank	HPCG (Pflop/s)	Fraction of Peak
1	DOE/SC/ORNL USA	**Summit**, AC922, IBM POWER9 22C 3.7GHz, Dual-rail Mellanox FDR, NVIDIA Volta V100, IBM	2,397,824	148.60	1	2.926	1.5%
2	DOE/NNSA/LLNL USA	**Sierra**, S922LC, IBM POWER9 20C 3.1 GHz, Mellanox EDR, NVIDIA Volta V100, IBM	1,572,480	94.64	2	1.796	1.4%
3	RIKEN Advanced Institute for Computational Science Japan	**K computer**, SPARC64 VIIIfx 2.0GHz, Tofu interconnect, Fujitsu	705,024	10.51	18	0.603	5.3%
4	DOE/NNSA/LANL/SNL USA	**Trinity**, Cray XC40, Intel Xeon E5-2698 v3 16C 2.3GHz, Aries, Cray	979,072	20.16	6	0.546	1.3%
5	Natl. Inst. Adv. Industrial Sci. and Tech. (AIST) Japan	**ABCI**, PRIMERGY CX2570M4, Intel Xeon Gold 6148 20C 2.4GHz, Infiniband EDR, NVIDIA Tesla V100, Fujitsu	368,640	16.86	10	0.509	1.7%
6	Swiss National Supercomputing Centre (CSCS) Switzerland	**Piz Daint**, Cray XC50, Intel Xeon E5-2690v3 12C 2.6GHz, Cray Aries, NVIDIA Tesla P100 16GB, Cray	387,872	21.23	5	0.497	1.8%
7	National Supercomputing Center in Wuxi China	**Sunway** TaihuLight, Sunway MPP, SW26010 260C 1.45GHz, Sunway, NRCPC	10,649,600	93.02	3	0.481	0.4%
8	Korea Institute of Science and Technology Information Republic of Korea	**Nurion**, CS500, Intel Xeon Phi 7250 68C 563584C 1.4GHz, Intel Omni-Path, Intel Xeon Phi 7250, Cray	570,020	13.93	13	0.391	1.5%
9	Joint Center for Advanced High Performance Computing Japan	**Oakforest-PACS**, PRIMERGY CX600 M1, Intel Xeon Phi Processor 7250 68C 1.4GHz, Intel Omni-Path Architecture, Fujitsu	556,104	13.55	14	0.385	1.5%
10	DOE/SC/LBNL/NERSC USA	**Cori**, XC40, Intel Xeon Phi 7250 68C 1.4GHz, Cray Aries, Cray	622,336	14.02	12	0.355	1.3%



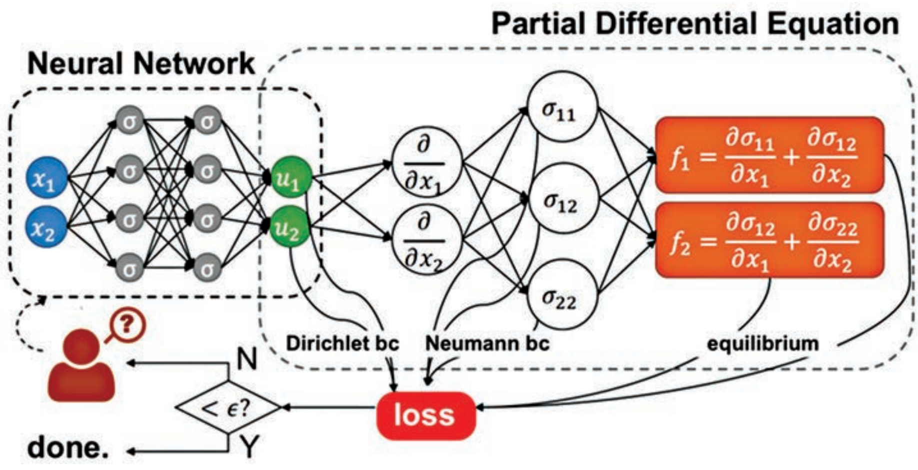
I like to speak about 'mathware', and isolate this from the software and the hardware. Mathware researchers should talk to their software and hardware colleagues, and discuss what is the best approach for a certain computational problem. Mathematicians always had quite a tight relation to software developers and computer scientists, but it will also become more important in future to speak to the hardware people. For example, one could make use of different representations of numbers: 16, 32 and 64 bit. First performing calculations in 16 bit representations is cheap, then one could step it up to 32 bit calculations, finally only having to perform few calculations with the most expensive 64 bit representation of numbers. This could be an ideal framework to carry out simulations in a much more efficient way. However, not all computers have the right semiconductor devices and chips on board to ease such transformations between 16, 32 and 64 bit representations. Hence, it is urgent to also speak to the hardware developers.

Another important development in recent years is the emergence of the field of *data science*, where abundant amounts of data are generated in many different areas, and the challenge is to extract useful information from this. I will not say too much about this, but it is clearly also an area where mathematicians should play a key role. The Department of Mathematics and Computer Science of TU/e founded the Data Science Centre Eindhoven (DSCE) around 2014, and now also has an educational track in data science. Data scientists are highly demanded in society and industry, but it is clear that extreme care needs to be taken to extract correct information from the data. Many of the methods used in the area of data science are already well known for many decades, which is also the reason why Stanford professor of statistics, David Donoho, wrote a somewhat cynical paper some years ago entitled '50 years of Data Science'.

The third development that will change scientific computing in the future is that of *artificial intelligence* (AI) and *machine learning* (ML). Just as with HPC, AI and ML were quite popular already a few decades ago, but the development of methods was only accelerated a few years ago. Deep neural networks were introduced in 2012, and since then have taken a giant leap forward. It has changed speech recognition on our phones enormously, and also had a big influence in many different areas. Also in the area of computational science, artificial intelligence, machine learning and artificial neural networks (ANN) started to be introduced, and clearly this is an extremely interesting development for the area of scientific computing. One of the simplest applications is to generate ANN models of parameters in the problem for which no physical models are available. Often, the physics of the underlying problem is known quite well, in the form of ordinary or partial differential equations, but such models may not be available for some parameters in these equations. I compare it to the situation we encountered at Philips Research: we simulated semiconductor devices by solving the set of drift-diffusion equations, but models for the recombination and mobility were provided by the electronic engineers and changed every year. The engineers determined these models by performing many simulations and experiments, thereby generating a lot of data from which they constructed a model by combining curve fitting and physical or engineering insight. We could do the same thing nowadays using the data to construct adequate neural networks describing the parameters in an accurate way. A problem that arises here is that one can mathematically prove properties of the parameters that should hold, which was not always the case for the man-made models. Clearly, such known properties must also hold for the artificial neural networks, and this is still a big challenge: how to produce ANN that have specified properties.

This is a general and very important question, and in fact is referring back to mimetic methods again: how can we develop artificial neural networks that retain certain properties of the underlying physical system? In other words, we would like to develop mimetic neural networks! An additional advantage is that much less data are needed, and on can have more confidence of reliable results also in case on performs simulations that are out of the range of training data. One attempt in this direction is provided by the so-called physics informed neural networks, better known as PINNs. These have been developed by the group of George Karniadakis of Brown University in the USA. A sketch of how they work is provided in the figure below.

As can be seen here, the partial differential equations are part of a loss function, and this means that they will not be solved accurately. Stated differently, the set of differential equations will not be solved, leading to an error that can sometimes be quite substantial, depending on the other components of the loss function.



In our UNRAVEL project, an NWO XL project, we are researching other ways of designing physics informed neural networks, with the explicit demand that the underlying differential equations are satisfied and vital properties of solutions are retained. A very interesting approach, which already originates from work within Philips Research, is that of designing truly dynamic neural networks. Often, for the dynamic case, so-called recurrent networks are designed. Our dynamic neural networks are truly dynamic, with dynamic actions in the neurons of the network. This approach has enormous potential in our opinion, and also with my team at TU Munich we will explore this further.

Mathematics can and should play a prominent role within the world of artificial intelligence, and to stress this, I always state that

> "Real intelligence is needed to make artificial intelligence work".

The future needs computational science and engineering, blending data driven and physics-based perspectives. The combination of scientific computing and machine learning constitutes a new field of research, termed scientific machine learning. Scientific machine learning has been taking the academic world by storm as an interesting blend of traditional scientific modelling with machine learning methodologies like deep learning. While traditional deep learning methodologies have had difficulties with scientific issues like stiffness, interpretability, and enforcing physical constraints, this blend with numerical analysis and differential equations has evolved into a field of research with new methods, architectures, and algorithms which overcome these problems while adding the data-driven automatic learning features of modern deep learning. Many successes have already been found, with tools like physics-informed neural networks, universal differential equations, deep backward stochastic differential equation solvers for high dimensional partial differential equations, and neural surrogates showcasing how deep learning can greatly improve scientific modelling practice. Mathematics will be essential in addressing the challenges that we encounter in the rapidly evolving field of scientific machine learning. In this context, it is really great that the national initiative 'AI and Mathematics' (AIM) has been set up. It emphasizes that mathematicians are very interested in participating in the world of artificial intelligence and machine learning, and I am convinced of a very essential role for mathematics in these fields.

All of the foregoing is necessary to address industrial challenges in the future. The high demand for extremely accurate models that can simulate processes and products in real time or even beyond real time necessitates the solution of coupled systems of equations. Here we use the terminology of *digital twins*: creating a virtual copy of the real product or process that is running in parallel, is being fed with data from sensors, and able to predict failures and recommend precautions to be taken. Huge savings can be obtained if such digital twins are used, but it will take some time before we will see true digital twins in industry. Companies like Siemens are heavily investing in digital twinning, acquiring software companies with software for different aspects such as electromagnetic, thermal, mechanical behaviour, just to make sure that every aspect is included.

The future of scientific computing will be bright in all of the aforementioned fields: high performance computing, data science and artificial intelligence. I fully agree with the statement of Karen Willcox, esteemed colleague from the Oden Institute in Texas: "It is such an exciting time to be a computational scientist. The field is in the midst of a tremendous convergence of technologies that generate unprecedented system data and enable automation, algorithms that let users process massive amounts of data and run predictive simulations that drive key decisions, and the computing power that makes these algorithms feasible at scale for complex systems and in real-time or in situ settings."

Mathematics for a better world

Besides the research carried out and described in the foregoing, I also played a role in various organisations, both at the national and international level, to promote mathematics in general, and also mathematics for applications. The impression people have about mathematics and mathematicians is not always positive, often mathematics is viewed as a rather difficult subject and one often does not have any idea about its usefulness. Mathematics learned in secondary schools is seen as a necessary thing that one needs to do, but can forget about soon after leaving school. It is important to counter this misperception, and show that mathematics is vital in our complex world with many grand challenges.

At the national level, there is the Dutch Platform for Mathematics, supported by NWO and all mathematical institutes. Several reports have been produced since the start in 2010, including the booklet on success stories mentioned before, and a Deloitte report detailing the value of mathematics for the Dutch economy. The latter contains stunning figures, indicating the extreme importance of mathematics for society, as mentioned before. Similar reports have been produced in the UK, France and Spain, with corresponding figures. The exhibition *Imaginary* has been touring the country in 2016–2017 and in 2022–2023, showing the beauty and the power of mathematics. And many other initiatives are undertaken to demonstrate the unexpected usefulness of mathematics.

At the European level, I have been involved in the European Consortium of Mathematics for Industry (ECMI) and EU-MATHS-IN, the European Service Organisation for Mathematics in Industry and Innovation. Both organisations have many researchers on board that work with industry, educate students to have careers in industry and lobby for mathematics at the European level. Also at this level, it is rather difficult to convince especially policy makers of the extremely important role of mathematics. Mathematical methods are often considered tools, like a hammer or a screw driver, that one can pick from the mathematical toolbox and just use. In my inaugural speech, I already indicated that this is not the ideal situation. It is much better to involve mathematicians from the start, they know the methods and their modifications inside out, and are in a much better position to judge which methods should be used or what needs to be done. Mathematicians have a very special way of thinking, often in an abstract way, and this is what is needed to successfully address challenges. Besides this, mathematics is a very versatile science. Methods that have been developed for a certain application can often be abstracted in such a way that application to completely different problems is possible. Often, this is unexpected. Why would a method developed for electronic circuit simulation, be useful for a mechanical or civil engineering problem? Mathematicians have the oversight and insight to judge such situations and suggest the use or further development of methods from entirely different fields.

Since 2019 I am an officer-at-large in the worldwide organisation for industrial and applied mathematics, ICIAM, and, in fact, its next president from 1 October 2023. Yes, indeed, a few months after my (formal) retirement! I would like to continue the work we did on the European level and take it to the larger scale, as it is evident that we can help each other to bring mathematics to the forefront and stress its importance for important challenges in the world like climate change and energy transition. During the recent SIAM conference on Computational Science and Engineering in the RAI Amsterdam, I organised a public evening with the title 'The role of mathematics in solving the world's main challenges'. The presentations can be viewed on YouTube (http://tinyurl.com/5b875efj), showing impressive examples of how mathematics can contribute to a better understanding of challenges and aid in matching the high demands that society and industry have nowadays.

I am coming to the end of my valedictory lecture. Especially in the last two decades, since my appointment at TU/e, I have considered it my mission to highlight the foregoing ideas and convince researchers and policy makers of the extreme value of mathematics for addressing societal and industrial challenges. In this context, it is adequate to quote the famous mathematical physicist Eugene Wigner, Nobel prize winner for physics in 1963, who said: "The unreasonable efficiency of mathematics in science is a gift we neither understand nor deserve." Let's not worry about the fact we may not understand or deserve it, instead let's use this gift! I am convinced that mathematics can contribute to a better world in many ways, and I hope that many will join the mission to convince everyone about this fact. It may seem a mission impossible, but I am sure in the end it will turn out to be a mission possible!

Closing words and thanks

Looking back on my life, a feeling of great gratitude prevails. I come from a warm catholic nest, where respect for my surroundings was taught and where I was encouraged to have a mission in life. At the Catholic University in Nijmegen, nowadays Radboud University, I had excellent and inspiring teachers such as Arnoud van Rooij and Ronald Kortram, and studying mathematics was a sincere pleasure. With John Miller, I had the privilege of having had a very inspiring teacher for the young field of numerical mathematics, later colleague and friend. At Philips, I was able to continue doing mathematics every day, thanks to the inspiring working environment in the Mathematical Software Group created by Simon Polak. He also taught me the pleasures in life of a researcher, going to conferences (and great hotels), and meeting many interesting people.

After my appointment at TU/e, I have thoroughly enjoyed spending my professional career in the field of scientific computing for industry and I have many to thank for that. I became known to colleagues as the traveling mathematician and an expert in food in many places in the world. (A book entitled *The traveling mathematician* will appear sometime after my retirement, devoted to Marcello Anile, a traveling mathematician from Sicily who unfortunately passed away far too young). The longer and many short trips were a tremendous enrichment of my life and work, sometimes also for family members. Organising symposia and conferences, collaborating in consortia, and all kinds of board work associated with that, has led to much satisfaction. Being allowed to transfer knowledge to students and seeing them grow into accomplished researchers and mathematicians who put their qualities for the benefit of industry, has also been a fascinating task.

I have many to thank for this fantastic professional career. In the first place, TU/e for trusting me to develop the field, for providing an excellent research and social environment, and giving the academic freedom to pursue interesting topics together with many nice people. I'm deeply indebted to all the academic and support staff of the Department of Mathematics and Computer Science and, in particular, to my colleagues in the Centre for Analysis, Scientific Computing and Applications. It is abbreviated CASA, and it has really felt as a home to me. I would like to thank the people who worked with me for their commitment and the fine cooperation all these years. I would also like to thank my colleagues in The Netherlands, in Europe and beyond for the very pleasant collaborations and aid in the joint mission we have.

Ik heb gezegd.

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