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Proof by example Portraits of women in Dutch mathematics

Assia Mahboubi

Assia Mahboubi is a tenured researcher at the Institut national de recherche en sciences et technologies du numérique (Inria), Nantes, France, and endowed professor in the Algebra and Number Theory section of the Vrije Universiteit Amsterdam. On 26 April she gave her inaugural lecture ‘Explore, abstract, compute, verify, can we do (all) mathematics by computer?’.

Thank you for accepting the interview for the NAW. I would like to begin by discussing the topic of your inaugural lecture. Can mathematics be done by computer? What are the main challenges involved?

“Certainly, mathematics can be done using computers. However, there are different aspects to consider. One aspect involves using computers for large calculations and incorporating them into proofs. Some skeptics argue that this approach undermines the true understanding of a proof, as they believe mathematics should be separate from calculation algorithms. Another challenge is the trustworthiness of computer-based calculations. While experts are sometimes able to validate their output, computer systems themselves cannot be fully trusted. However, there have been significant advancements in formal methods for ensuring program correctness in computer science, and there is hope that similar techniques can be applied to mathematical software.”

What kind of problems are you currently working on?

“My current research focuses on various aspects of mathematics and its formal representation. One area of my research

explores different foundations for formalizing mathematics on computers. This involves using computers not only for calculations but also to represent and check formal proofs. This field raises theoretical questions and opens up new avenues for mathematical investigations. I am also interested in developing tools that can assist mathematicians in ensuring the correctness of their calculations, particularly in the realm of symbolic computation. This involves designing theoretical tools to address the proofs suggested by such applications.”

How can we help mathematicians compute in a safe way and connect their proofs on paper to the computational parts?

“This is a significant challenge. Currently, my work involves developing tools and methods for mathematicians to ensure the correctness of their calculations and connect them with formal proofs. One approach is to use proof assistants, such as Coq, which is based on the same logical foundations as Lean. I find it interesting to have different systems based on the same theoretical foundations, as it allows for experimentation and different approaches.”



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Do different mathematical fields have different needs?

“Certain mathematical fields have specific software programs that are more suitable for their needs. For example, there are specialized software libraries for symbolic computations, algebra, and analysis. However, when it comes to formal proof assistants, the focus is more on mathematics in general rather than specific fields. While there have been remarkable developments in certain specialized areas, the current systems are still in a preliminary stage and cannot be considered fully specialized.”

Is there any research result that you're particularly proud of or enthusiastic about?

“One result that stands out is a project I worked on with a former PhD student and colleague Thomas Sibut-Pinote [1]. We studied a result in number theory called the Apéry theorem, which addresses the irrationality of certain values of the Riemann zeta function $\zeta(s)$. The transcendence, or even the irrationality, of the values of the ζ function at odd integers is a challenging problem in number theory, and this result asserts that $\zeta(3)$ is irrational. We were able to provide a formally verified proof using computer algebra tools and a proof assistant, combining the strengths of both systems. It was a fun project that demonstrated the interaction between computer algebra and proof assistants.”

Could you share how you became interested in mathematics in the first place?

“I have always had a fondness for mathematics. Despite coming from a non-scientific family, I found it incredibly enjoyable. From a young age, I would solve math problems and engage in exercises from

books. However, I wasn't sure if I wanted to pursue it as a career. In high school, I also developed an interest in philosophy, so I had to choose between the two. My parents believed that pursuing a career in science would guarantee a job, so they advised me to choose that path and continue doing math as a hobby. At that time, I wasn't even aware of research in mathematics; my only knowledge was that of becoming a teacher. While studying in France, I attended a popular science talk by the mathematician Alain Connes, recommended by my math teacher. That was the first time I heard a mathematician speak, and it made me realize that being a mathematician was a viable career option. From that point on, I decided to pursue mathematics seriously.”

What attracted you to questions related to foundations and formalization of mathematics?

“Initially, my focus was primarily on computer algebra, particularly within the context of formal verification. The idea was to design algorithms that could produce certificates for algebraic problems, which could be verified later. When possible, outputs then become easier to check than to produce. However, I was part of a team where my advisor was more interested in computer algebra, while others were focused on logic. As a result, I began working in both areas and became gradually interested in the questions related to theoretical computer science.”

Is there anything about being a mathematician that you find challenging?

“When it comes to the actual process of doing mathematics, I find it enjoyable and fulfilling. However, like many others, I struggle with the surrounding activities, such as logistics, research management,

grant proposals, and administrative tasks. These activities tend to consume more and more time, leaving less time for actual mathematics. Moreover, as a woman in France, there are additional challenges such as quotas and gender-related biases that can affect the working environment.”

What do you think can be done to improve the working environment and promote diversity in mathematics?

“Simply increasing quotas is not an effective solution. Instead, we should aim for a sensible approach that respects the existing proportion of women and underrepresented groups in different communities. It's important to involve women in mathematics without imposing artificial quotas. Additionally, measures can be taken to support individuals in balancing work and family life, such as providing assistance for childcare and facilitating participation in events. These measures benefit not only women but also promote diversity more broadly.”

And finally, on a positive note, what do you enjoy most about being a mathematician?

“What I enjoy the most about being a mathematician is the opportunity for continuous learning. Mathematics is an endless game, and there is always something new and surprising to discover. Each area has its unique techniques, objects, and nature that make it fascinating. The joy of learning in mathematics remains intact even after years of studying. It's a privilege to have the opportunity to engage in such a fulfilling pursuit.” ❖

Reference

- 1 A. Mahboubi and T. Sibut-Pinote, A formal proof of the irrationality of $\zeta(3)$, *Log. Methods Comput. Sci.* 17(1) (2021).